Mathematica 11.3 Integration Test Results

Test results for the 27 problems in "4.6.7 (d trig)^m (a+b (c csc)^n)^p.m"

Problem 2: Result more than twice size of optimal antiderivative.

$$\int \left(a+b\,\mathrm{Csc}[c+d\,x]^2\right)^3 \, dx$$

Optimal (type 3, 74 leaves, 4 steps):

$$a^3 x-\frac{b\left(3 a^2+3 a b+b^2\right)\mathrm{Cot}[c+d\,x]}{d}-\frac{b^2\,(3 a+2 b)\,\mathrm{Cot}[c+d\,x]^3}{3 d}-\frac{b^3\,\mathrm{Cot}[c+d\,x]^5}{5 d}$$

Result (type 3, 266 leaves):

$$\frac{8 b^3 \mathrm{Cos}[c+d\,x]\left(a+b\,\mathrm{Csc}[c+d\,x]^2\right)^3 \mathrm{Sin}[c+d\,x]}{5 d\,(-a-2 b+a\,\mathrm{Cos}[2\,(c+d\,x)])^3}+\left(8\left(15 a b^2 \mathrm{Cos}[c+d\,x]+4 b^3 \mathrm{Cos}[c+d\,x]\right)\left(a+b\,\mathrm{Csc}[c+d\,x]^2\right)^3 \mathrm{Sin}[c+d\,x]^3\right)\Big/\left(15 d\,(-a-2 b+a\,\mathrm{Cos}[2\,(c+d\,x)])^3\right)+\left(8\left(45 a^2 b\,\mathrm{Cos}[c+d\,x]+30 a b^2 \mathrm{Cos}[c+d\,x]+8 b^3 \mathrm{Cos}[c+d\,x]\right)\left(a+b\,\mathrm{Csc}[c+d\,x]^2\right)^3 \mathrm{Sin}[c+d\,x]^5\right)\Big/\left(15 d\,(-a-2 b+a\,\mathrm{Cos}[2\,(c+d\,x)])^3\right)-\frac{8 a^3\,(c+d\,x)\left(a+b\,\mathrm{Csc}[c+d\,x]^2\right)^3 \mathrm{Sin}[c+d\,x]^6}{d\,(-a-2 b+a\,\mathrm{Cos}[2\,(c+d\,x)])^3}$$

Problem 3: Result more than twice size of optimal antiderivative.

$$\int \left(a+b\,\mathrm{Csc}[c+d\,x]^2\right)^2 \, dx$$

Optimal (type 3, 41 leaves, 4 steps):

$$a^2 x-\frac{b\,(2 a+b)\,\mathrm{Cot}[c+d\,x]}{d}-\frac{b^2\,\mathrm{Cot}[c+d\,x]^3}{3 d}$$

Result (type 3, 83 leaves):

$$-\left(\left(4\left(a+b\,\mathrm{Csc}[c+d\,x]^2\right)^2\left(-3 a^2\,(c+d\,x)+b\,\mathrm{Cot}[c+d\,x]\left(6 a+2 b+b\,\mathrm{Csc}[c+d\,x]^2\right)\right)\mathrm{Sin}[c+d\,x]^4\right)\Big/\left(3 d\,(a+2 b-a\,\mathrm{Cos}[2\,(c+d\,x)])^2\right)\right)$$

Problem 8: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\left(a+b\,\mathrm{Csc}[c+d\,x]^2\right)^4}\,dx$$

Optimal (type 3, 204 leaves, 7 steps):

$$\frac{x}{a^4}+\frac{\sqrt{b}\,\left(35\,a^3+70\,a^2\,b+56\,a\,b^2+16\,b^3\right)\,\mathrm{ArcTan}\left[\frac{\sqrt{b}\,\mathrm{Cot}[c+d\,x]}{\sqrt{a+b}}\right]}{16\,a^4\,\left(a+b\right)^{7/2}\,d}+\frac{b\,\mathrm{Cot}[c+d\,x]}{6\,a\,\left(a+b\right)\,d\,\left(a+b+b\,\mathrm{Cot}[c+d\,x]^2\right)^3}+\frac{b\,\left(11\,a+6\,b\right)\,\mathrm{Cot}[c+d\,x]}{24\,a^2\,\left(a+b\right)^2\,d\,\left(a+b+b\,\mathrm{Cot}[c+d\,x]^2\right)^2}+\frac{b\,\left(19\,a^2+22\,a\,b+8\,b^2\right)\,\mathrm{Cot}[c+d\,x]}{16\,a^3\,\left(a+b\right)^3\,d\,\left(a+b+b\,\mathrm{Cot}[c+d\,x]^2\right)}$$

Result (type 3, 410 leaves):

$$\frac{\left(c+d\,x\right)\,\left(-a-2\,b+a\,\mathrm{Cos}\left[2\,\left(c+d\,x\right)\right]\right)^4\,\mathrm{Csc}[c+d\,x]^8}{16\,a^4\,d\,\left(a+b\,\mathrm{Csc}[c+d\,x]^2\right)^4}-\left(\sqrt{b}\,\left(35\,a^3+70\,a^2\,b+56\,a\,b^2+16\,b^3\right)\,\mathrm{ArcTan}\left[\frac{\sqrt{a+b}\,\mathrm{Tan}[c+d\,x]}{\sqrt{b}}\right]\,\left(-a-2\,b+a\,\mathrm{Cos}\left[2\,\left(c+d\,x\right)\right]\right)^4\,\mathrm{Csc}[c+d\,x]^8\right)\Big/\left(256\,a^4\,\left(a+b\right)^{7/2}\,d\,\left(a+b\,\mathrm{Csc}[c+d\,x]^2\right)^4\right)-\frac{b^3\,\left(-a-2\,b+a\,\mathrm{Cos}\left[2\,\left(c+d\,x\right)\right]\right)\,\mathrm{Csc}[c+d\,x]^8\,\mathrm{Sin}\left[2\,\left(c+d\,x\right)\right]}{24\,a^3\,\left(a+b\right)\,d\,\left(a+b\,\mathrm{Csc}[c+d\,x]^2\right)^4}+\left(\left(-a-2\,b+a\,\mathrm{Cos}\left[2\,\left(c+d\,x\right)\right]\right)^3\,\mathrm{Csc}[c+d\,x]^8\,\left(-87\,a^2\,b\,\mathrm{Sin}\left[2\,\left(c+d\,x\right)\right]-116\,a\,b^2\,\mathrm{Sin}\left[2\,\left(c+d\,x\right)\right]-44\,b^3\,\mathrm{Sin}\left[2\,\left(c+d\,x\right)\right]\right)\right)\Big/\left(768\,a^3\,\left(a+b\right)^3\,d\,\left(a+b\,\mathrm{Csc}[c+d\,x]^2\right)^4\right)+\left(\left(-a-2\,b+a\,\mathrm{Cos}\left[2\,\left(c+d\,x\right)\right]\right)^2\,\mathrm{Csc}[c+d\,x]^8\,\left(-19\,a\,b^2\,\mathrm{Sin}\left[2\,\left(c+d\,x\right)\right]-14\,b^3\,\mathrm{Sin}\left[2\,\left(c+d\,x\right)\right]\right)\right)\Big/\left(192\,a^3\,\left(a+b\right)^2\,d\,\left(a+b\,\mathrm{Csc}[c+d\,x]^2\right)^4\right)$$

Problem 9: Result more than twice size of optimal antiderivative.

$$\int \left(a+b\,\mathrm{Csc}[c+d\,x]^2\right)^{5/2}\,dx$$

Optimal (type 3, 167 leaves, 8 steps):

$$-\frac{a^{5/2}\,\mathrm{ArcTan}\left[\frac{\sqrt{a}\,\mathrm{Cot}[c+d\,x]}{\sqrt{a+b+b\,\mathrm{Cot}[c+d\,x]^2}}\right]}{d}-\frac{\sqrt{b}\,\left(15\,a^2+10\,a\,b+3\,b^2\right)\,\mathrm{ArcTanh}\left[\frac{\sqrt{b}\,\mathrm{Cot}[c+d\,x]}{\sqrt{a+b+b\,\mathrm{Cot}[c+d\,x]^2}}\right]}{8\,d}-\frac{b\,\left(7\,a+3\,b\right)\,\mathrm{Cot}[c+d\,x]\,\sqrt{a+b+b\,\mathrm{Cot}[c+d\,x]^2}}{8\,d}-\frac{b\,\mathrm{Cot}[c+d\,x]\,\left(a+b+b\,\mathrm{Cot}[c+d\,x]^2\right)^{3/2}}{4\,d}$$

Result (type 3, 396 leaves):

$$-\left(\left(\left(-4 a^3-15 a^2 b-10 a b^2-3 b^3\right) \text{ArcTanh}\left[\frac{\sqrt{2} \sqrt{-b} \text{Cos}[c+d x]}{\sqrt{-a-2 b-a \text{Cos}\left[2 \left(-c+\frac{\pi}{2}-d x\right)\right]}}\right]\right.\right.$$
$$\left.\left.\left(a+b \text{Csc}[c+d x]^2\right)^{5/2} \text{Sin}[c+d x]^5\right) \Big/ \left(\sqrt{2} \sqrt{-b} d \left(-a-2 b+a \text{Cos}[2 (c+d x)]\right)^{5/2}\right)\right)+$$
$$\left(\left(a+b \text{Csc}[c+d x]^2\right)^{5/2} \left(-\frac{3}{2} \left(3 a b \text{Cos}[c+d x]+b^2 \text{Cos}[c+d x]\right) \text{Csc}[c+d x]^2-\right.\right.$$
$$\left.\left.b^2 \text{Cot}[c+d x] \text{Csc}[c+d x]^3\right) \text{Sin}[c+d x]^5\right) \Big/ \left(d \left(-a-2 b+a \text{Cos}[2 (c+d x)]\right)^2\right)+$$
$$\left(4 a^3 \left(a+b \text{Csc}[c+d x]^2\right)^{5/2} \left(-\frac{\text{ArcTanh}\left[\frac{\sqrt{2} \sqrt{-b} \text{Cos}[c+d x]}{\sqrt{-a-2 b+a \text{Cos}[2 (c+d x)]}}\right]}{\sqrt{2} \sqrt{-b}}+\right.\right.$$
$$\left.\frac{\sqrt{2} \text{Log}\left[\sqrt{2} \sqrt{a} \text{Cos}[c+d x]+\sqrt{-a-2 b+a \text{Cos}[2 (c+d x)]}\right]}{\sqrt{a}}\right)$$
$$\left.\text{Sin}[c+d x]^5\right) \Big/ \left(d \left(-a-2 b+a \text{Cos}[2 (c+d x)]\right)^{5/2}\right)$$

Problem 12: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{a+b \text{Csc}[c+d x]^2}} \, dx$$

Optimal (type 3, 39 leaves, 3 steps):

$$-\frac{\text{ArcTan}\left[\frac{\sqrt{a} \text{Cot}[c+d x]}{\sqrt{a+b \text{Csc}[c+d x]^2}}\right]}{\sqrt{a} d}$$

Result (type 3, 98 leaves):

$$-\left(\left(\sqrt{-a-2 b+a \text{Cos}[2 (c+d x)]} \text{Csc}[c+d x] \text{Log}\left[\right.\right.\right.$$
$$\left.\left.\left.\sqrt{2} \sqrt{a} \text{Cos}[c+d x]+\sqrt{-a-2 b+a \text{Cos}[2 (c+d x)]}\right]\right) \Big/ \left(\sqrt{2} \sqrt{a} d \sqrt{a+b \text{Csc}[c+d x]^2}\right)\right)$$

Problem 16: Result more than twice size of optimal antiderivative.

$$\int \left(1+\mathrm{Csc}[x]^2\right)^{3/2} \, dx$$

Optimal (type 3, 44 leaves, 6 steps):

$$-2\,\mathrm{ArcSinh}\left[\frac{\mathrm{Cot}[x]}{\sqrt{2}}\right]-\mathrm{ArcTan}\left[\frac{\mathrm{Cot}[x]}{\sqrt{2+\mathrm{Cot}[x]^2}}\right]-\frac{1}{2}\,\mathrm{Cot}[x]\,\sqrt{2+\mathrm{Cot}[x]^2}$$

Result (type 3, 94 leaves):

$$\left(\left(1+\mathrm{Csc}[x]^2\right)^{3/2}\left(-4\sqrt{2}\,\mathrm{ArcTan}\left[\frac{\sqrt{2}\,\mathrm{Cos}[x]}{\sqrt{-3+\mathrm{Cos}[2x]}}\right]+\sqrt{-3+\mathrm{Cos}[2x]}\,\mathrm{Cot}[x]\,\mathrm{Csc}[x]-2\sqrt{2}\,\mathrm{Log}\left[\sqrt{2}\,\mathrm{Cos}[x]+\sqrt{-3+\mathrm{Cos}[2x]}\right]\right)\mathrm{Sin}[x]^3\right)\Big/\left(-3+\mathrm{Cos}[2x]\right)^{3/2}$$

Problem 17: Result more than twice size of optimal antiderivative.

$$\int \sqrt{1+\mathrm{Csc}[x]^2} \, dx$$

Optimal (type 3, 28 leaves, 5 steps):

$$-\mathrm{ArcSinh}\left[\frac{\mathrm{Cot}[x]}{\sqrt{2}}\right]-\mathrm{ArcTan}\left[\frac{\mathrm{Cot}[x]}{\sqrt{2+\mathrm{Cot}[x]^2}}\right]$$

Result (type 3, 68 leaves):

$$\frac{1}{\sqrt{-3+\mathrm{Cos}[2x]}}\sqrt{2}\,\sqrt{1+\mathrm{Csc}[x]^2}\left(\mathrm{ArcTan}\left[\frac{\sqrt{2}\,\mathrm{Cos}[x]}{\sqrt{-3+\mathrm{Cos}[2x]}}\right]+\mathrm{Log}\left[\sqrt{2}\,\mathrm{Cos}[x]+\sqrt{-3+\mathrm{Cos}[2x]}\right]\right)\mathrm{Sin}[x]$$

Problem 18: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{1+\mathrm{Csc}[x]^2}} \, dx$$

Optimal (type 3, 16 leaves, 3 steps):

$$-\mathrm{ArcTan}\left[\frac{\mathrm{Cot}[x]}{\sqrt{2+\mathrm{Cot}[x]^2}}\right]$$

Result (type 3, 49 leaves):

$$-\frac{\sqrt{-3+\mathrm{Cos}[2x]}\,\mathrm{Csc}[x]\,\mathrm{Log}\left[\sqrt{2}\,\mathrm{Cos}[x]+\sqrt{-3+\mathrm{Cos}[2x]}\right]}{\sqrt{2}\,\sqrt{1+\mathrm{Csc}[x]^2}}$$

Problem 26: Result more than twice size of optimal antiderivative.

$$\int \sqrt{-1-\text{Csc}[x]^2}\, dx$$

Optimal (type 3, 33 leaves, 6 steps):

$$\text{ArcTan}\left[\frac{\text{Cot}[x]}{\sqrt{-2-\text{Cot}[x]^2}}\right]+\text{ArcTanh}\left[\frac{\text{Cot}[x]}{\sqrt{-2-\text{Cot}[x]^2}}\right]$$

Result (type 3, 70 leaves):

$$\frac{1}{\sqrt{-3+\text{Cos}[2\,x]}}\sqrt{2}\,\sqrt{-1-\text{Csc}[x]^2}\left(\text{ArcTan}\left[\frac{\sqrt{2}\,\text{Cos}[x]}{\sqrt{-3+\text{Cos}[2\,x]}}\right]+\text{Log}\left[\sqrt{2}\,\text{Cos}[x]+\sqrt{-3+\text{Cos}[2\,x]}\right]\right)\text{Sin}[x]$$

Problem 27: Result more than twice size of optimal antiderivative.

$$\int \frac{1}{\sqrt{-1-\text{Csc}[x]^2}}\, dx$$

Optimal (type 3, 18 leaves, 3 steps):

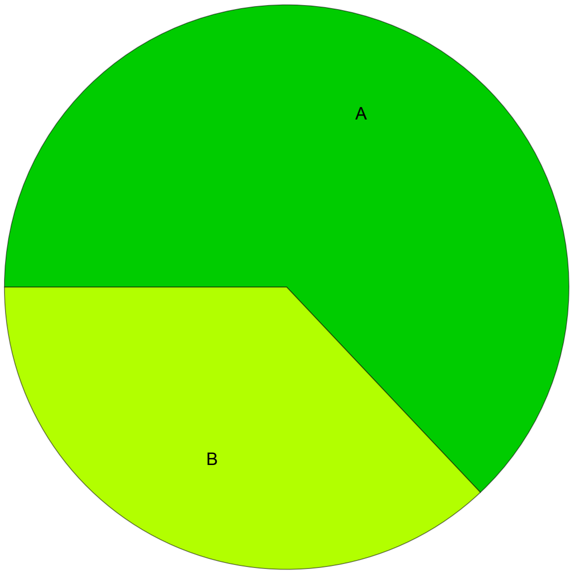
$$-\text{ArcTanh}\left[\frac{\text{Cot}[x]}{\sqrt{-2-\text{Cot}[x]^2}}\right]$$

Result (type 3, 51 leaves):

$$-\frac{\sqrt{-3+\text{Cos}[2\,x]}\,\text{Csc}[x]\,\text{Log}\left[\sqrt{2}\,\text{Cos}[x]+\sqrt{-3+\text{Cos}[2\,x]}\right]}{\sqrt{2}\,\sqrt{-1-\text{Csc}[x]^2}}$$

Summary of Integration Test Results

27 integration problems



A - 17 optimal antiderivatives

B - 10 more than twice size of optimal antiderivatives

C - 0 unnecessarily complex antiderivatives

D - 0 unable to integrate problems

E - 0 integration timeouts